Math 8174: Homework 6

Due April 13, 2009

1. Find all the irreducible $U_3(\mathbb{F}_5)$-modules for

$$U_3(\mathbb{F}_5) = \left\{ \begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \;\middle|\; a, b, c \in \mathbb{F}_5 \right\}.$$

   Hint: Note that the group generated by the last column is abelian and is acted on by the group generated by a's.

2. We could have proved our classification theorem of $G(r,1,n)$-modules in the same way we proved the analogous result for $G(1,1,n)$. The key step to this approach is finding the correct analogue of Murphy-Jucys elements.

   (a) Find an analogue to the Murphy-Jucys elements for $G(r,1,n)$. That is, find $m_k, j_l \in \mathbb{C}G(r,1,n)$, such that if λ is an r-tuple of partitions and T a tableau of shape λ, then

   $$m_k v_T = c(T(k)) v_T \qquad \text{and} \qquad j_l v_T = e^{2\pi i \mathrm{loc}_T(j)/r} v_T.$$

   (b) Explain how these elements imply that for $\lambda, \mu \in \hat{G}_1$, we have $G(r,1,n)^\lambda \cong G(r,1,n)^\mu$ if and only if $\lambda = \mu$.

3. A character $\chi : G \to \mathbb{C}$ is called *real-valued* if $\chi(g) \in \mathbb{R}$ for all $g \in G$.

   (a) For which of the $G(r,1,n)$ are all characters real-valued?

   (b) Let $\chi : G \to \mathbb{C}$ be a real-valued, irreducible character, let $\psi : G \to \mathbb{C}$ be an irreducible character, and let $H \subseteq G$ be a subgroup. Show that the module $V_\chi \otimes V_\psi$ contains the trivial module of G if and only if $\psi = \chi$. (See midterm for a definition of $V_\chi \otimes V_\psi$).