Math 8174: Homework 5

Due March 11–13, 2009

1. Let D_8 be the dihedral group of order 8 and let Q_8 be the quaternion group of order 8.

   (a) Show that D_8 and Q_8 have the same character table (ie. find the character table of Q_8).

   (b) Conclude that the character table does not give the number of involutions of a group.

2. Let $\pi : G \to H$ be a surjective group homomorphism. For a character $\chi : G \to \mathbb{C}$, define

$$\mathrm{Def}_H^G(\chi) : H \to \mathbb{C}$$

   by

$$\langle \psi_\pi, \chi \rangle_{C(G)} = \langle \psi, \mathrm{Def}_H^G(\chi) \rangle_{C(H)}.$$

   (a) Find a formula for the value $\mathrm{Def}_H^G(\chi)(h)$ for $h \in H$.

   (b) Show that $\mathrm{Def}_H^G(\chi)$ is a character by finding a module with respect to which it is the trace.

   Hint: For (b), consider multiplying the module corresponding to χ by an appropriate idempotent.