1. Consider the symmetric group $S_4$ and the dihedral group $D_8$. For each group $G$

   (a) Give examples of two nonequivalent and nontrivial representations $\rho$ and $\tau$ (be sure to show they are not equivalent),
   
   (b) Construct the corresponding $G$-modules $V_\rho$ and $V_\tau$,
   
   (c) Decide whether the modules are reducible,
   
   (d) Change bases in the module $V_\rho$ and give the new corresponding representation $\rho' : G \to GL_n(\mathbb{C})$.

2. Show that if $\rho : G \to GL(V)$ is a degree one representation, then $G/\ker(\rho)$ is an abelian group.

3. Let $GL_2(\mathbb{F}_q)$ be the general linear group of rank 2 with entries in the field $\mathbb{F}_q$ with $q$ elements. Consider the subalgebra of $\mathbb{C}GL_2(\mathbb{F}_q)$ given by

   $\mathcal{H}_2(q) = e_B \mathbb{C}GL_2(\mathbb{F}_q)e_B$, where $e_B = \frac{1}{q} \sum_{r,s \in \mathbb{F}_q^\times} \sum_{t \in \mathbb{F}_q} \begin{pmatrix} r & t \\ 0 & s \end{pmatrix}$.

   (This is the Iwahori-Hecke algebra $\mathcal{H}_2(q)$).

   (a) Find a basis for $\mathcal{H}_2(q)$.
   
   (b) Give formulas for multiplying basis elements.
   
   (c) Construct a nontrivial $\mathcal{H}_2(q)$-module that is not the regular module.