

## Math 8174: Homework 5

Due: November 3, 2010

1. We saw that one can obtain the roots  $\Phi$  of a Lie algebra from knowledge only of the Dynkin diagram  $\Gamma$ . Do this explicitly for the root system of type  $D_4$ .
2. Prove that the subspace  $M \subseteq \mathfrak{g} \oplus \mathfrak{g}'$  from the isomorphism theorem proof is in fact equal to  $\mathfrak{d}$  with the correct choices of  $e_\beta \in \mathfrak{g}^\beta$  and  $e_{\beta'} \in (\mathfrak{g}')^{\beta'}$ .
3. Show that for simple Lie algebras  $R^\vee$  is isomorphic to  $R$  unless  $\mathfrak{g}$  is of type  $B$  or  $C$  in which case  $R^\vee$  switches between the two.
4. Show that  $U(\mathfrak{g})$  has no zero divisors.