Math 6350: Homework 6

Due: Monday, December 10

A.

VI.1.1.1. Show that if S is symmetric about the real axis and our chosen point $a \in S$ is real, then the function f of the Riemann Mapping Theorem satisfies $\overline{f(z)} = f(\bar{z})$.

VI.1.1.2. What is the corresponding statement if S is symmetric around the point $a \in S$.

B.

(1) Find a closed form for the generating function of Fibbonacci numbers,

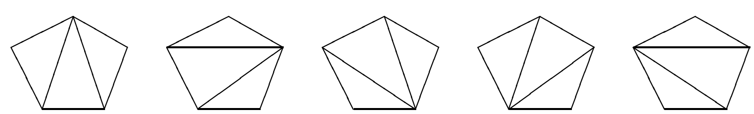
$$f_0 = 0, \qquad f_1 = 1, \qquad f_n = f_{n-1} + f_{n-2}.$$

Use partial fractions to find a closed form for f_n.

(2) The Catalan numbers $\{c_n\}_{n \geq 0}$ are given by

$$c_n = \text{The number of ways to triangulate a convex } (n+1)\text{-gon.}$$

For example, the $c_4 = 5$, since



(a) Show that the Catalan numbers satisfy the recursion,

$$c_n = \sum_{k=1}^{n-1} c_k c_{n-k}.$$

(b) Sue the recursion to show that if $C(z)$ is the ordinary generating function for $\{c_n\}$, then

$$C(z) = \frac{1 - \sqrt{1 - 4z}}{2}.$$

(c) What conclusions can you reach by applying the first and second principles of coefficient asymptotics?