

# Math 6350: Homework 1

Due: Friday, September 7

A. Let

$$D: \mathbb{C} \times \mathbb{C} \longrightarrow \mathbb{C} \\ (a, b) \mapsto \frac{2|a - b|}{\sqrt{(1 + |a|^2)(1 + |b|^2)}}.$$

- (1) Show that  $D$  defines a metric for  $\mathbb{C}$ .
- (2) Show that this metric is not complete.

B. For  $a \in \mathbb{C}$ ,

$$T_a(z) = \frac{z - a}{1 - \bar{a}z}, \quad \text{where } z \in \mathbb{C}.$$

- (1) Show that  $T_a$  maps

$$S^1 = \{z \in \mathbb{C} \mid |z| = 1\}$$

onto itself.

- (2) Show that  $T_a$  maps

$$\mathbb{D} = \{z \in \mathbb{C} \mid |z| \leq 1\}$$

onto itself.

C. Let

$$f(z) = \lim_{n \rightarrow \infty} \frac{z^n - 1}{z^n + 1}.$$

- (1) Find the domain of  $f$ .
- (2) Is it possible to extend  $f$  to all of  $\mathbb{C}$  in such a fashion that  $f$  is continuous?