Math 6250: Homework 3

1. Let D_{2r} be the dihedral group of order $2r$. Completely classify the irreducible D_{2r}-modules. Hints: All irreducible D_{2r}-modules have dimension ≤ 2. Treat r even and r odd separately.

2. Let $n \in \mathbb{Z}_{\geq 1}$. Let λ be a partition of n. Let

$$V^\lambda = \mathbb{C}\text{-span}\{v_T \mid T \text{ a standard tableau of shape } \lambda\}.$$

For $i \in \{1, 2, \ldots, n-1\}$, define

$$s_i v_T = \begin{cases} v_{s_i(T)}, & \text{if } s_i(T) \text{ is standard,} \\ v_T, & \text{otherwise.} \end{cases}$$

Is V^λ an S_n-module under this action? If so, what is its decomposition into irreducibles?

3. (8.6 from book) Let ${}_RM$ and $S = \mathrm{End}_R(M)$. If $e \in S$ is an idempotent, show that

$$\mathrm{Tr}_M(Me) = (Me)S \qquad \text{and} \qquad \mathrm{Rej}_M(Me) = l_M(Se).$$

4. (8.10 from book) Let $\mathfrak{b}_n$ be the ring of uppertriangular matrices over a field $\mathbb{F}$. Show that

   (a) $\mathrm{Tr}_{\mathfrak{b}_n}(\hat{\mathfrak{b}}_n) = \{a \in \mathfrak{b}_n \mid a_{ij} = 0, i \geq 2\}$,

   (b) $\mathrm{Rej}_{\mathfrak{b}_n}(\hat{\mathfrak{b}}_n) = \{a \in \mathfrak{b}_n \mid a_{ii} = 0, 1 \leq i \leq n\}$.

5. (9.12 from book) Show that a product $\prod_{\alpha \in \mathcal{I}} V^{(i)}$ with $V^{(i)} \in \hat{R}$ is not necessarily completely reducible.