

## Math 6140: Homework 10

1. 13.6: 6, 8, 11, 12
2. 14.1: 5, 6, 8, 10
3. Suppose  $\mathbb{K}/\mathbb{F}$  is Galois of degree  $p$  and suppose  $\mathbb{K} = \mathbb{F}(\alpha)$  with  $\alpha^p \in \mathbb{F}$ . Show that  $\mathbb{K}$  contains a primitive  $p$ th root of unity, and when  $p$  is prime  $\mathbb{F}$  has all of them.
4. Use  $e^{2\pi i/7} + e^{12\pi i/7}$  to find an explicit polynomial  $f(x) \in \mathbb{Q}[x]$  such that the Galois group of its splitting field over  $\mathbb{Q}$  has 3 elements.