Math 6140: Homework 9

1. 14.1: 5, 6, 8, 10

2. Suppose $\mathbb{K}/\mathbb{F}$ is Galois of degree p and suppose $\mathbb{K} = \mathbb{F}(\alpha)$ with $\alpha^p \in \mathbb{F}$. Show that $\mathbb{K}$ contains a primitive pth root of unity, and when p is prime $\mathbb{F}$ has all of them.

3. Use $e^{2\pi i/7} + e^{12\pi i/7}$ to find an explicit polynomial $f(x) \in \mathbb{Q}[x]$ such that the Galois group of its splitting field over $\mathbb{Q}$ has 3 elements.

4. 14.2: 17, 18