Math 4140: Homework 7

Due: March 9, 2011

Required

1. Let λ be a partition of n. Let

$$V^\lambda = \mathbb{C}\text{-span}\{v_T \mid T \text{ a standard tableau of shape } \lambda\}.$$

For $i \in \{1, 2, \ldots, n-1\}$, define

$$v_T s_i = \begin{cases} v_{(Ts_i)}, & \text{if } Ts_i \text{ is standard,} \\ v_T, & \text{otherwise} \end{cases}$$

Is V^λ an S_n-module under this action? If so, what is its decomposition into irreducibles?

2. (a) Classify the conjugacy classes of D_6.
   (b) Classify the conjugacy classes of D_8.
   (c) Generalize to get an indexing set for the conjugacy classes of D_{2n}.

3. Let G be a finite group, and let U and V be G-modules.

   (a) Find a function
   $$\mathrm{Hom}_G(U, V) \times G \longrightarrow \mathrm{Hom}_G(U, V)$$
   that makes $\mathrm{Hom}_G(U, V)$ a G-module.
   Hint: Compare with Homework 1, Problem 3.
   (b) What is the dimension of $\mathrm{Hom}_{D_6}(\mathbb{C}D_6, \mathbb{C}D_6)$?
   (c) Decompose $\mathrm{Hom}_{D_6}(\mathbb{C}D_6, \mathbb{C}D_6)$ into irreducible D_6-modules.

Recommended

Note that recommended problems come from our book. They have answers in the back (I will not grade them, though I am happy to talk about them).

1. Chapter 11: 3, 6
2. Chapter 12: 4, 6