Math 4140: Homework 6

Due: March 1, 2011

Required

1. If G is a nonabelian group of order 6, what are the possible dimensions of its irreducible modules? If the group has order 8? If the group is order 12? For each case, list all the possible combinations.

2. Let
$$Q_8 = \{\pm 1, \pm i, \pm j, \pm k\},$$
where
$$i^2 = j^2 = k^2 = -1, (-1)^2 = 1, (-1)a = -a = a(-1), a \in \{1, i, j, k\},$$
$$ij = -ji = k, jk = -kj = i, ki = -ik = j.$$
   (a) Find all the one-dimensional representations of Q_8.
   (b) How many additional irreducibles are there (and what is their dimension)?
   (c) Complete the classification of irreducible Q_8-modules.

3. An *idempotent* $e \in \mathbb{C}G$ is an element such that
$$e^2 = e.$$
   (a) Show that $\frac{1}{|G|}\sum_{g \in G} g \in \mathbb{C}G$ is an idempotent.
   (b) Suppose $V \subseteq \mathbb{C}G$ is a submodule. As in Maschke's Theorem, let $\pi : \mathbb{C}G \to \mathbb{C}G$ be a projection that is a G-module homomorphism such that $\mathbb{C}G = \ker(\pi) \oplus V$. Show that $(1\pi) \in \mathbb{C}G$ is an idempotent.
   (c) Conclude that for every submodule $V \subseteq \mathbb{C}G$, $V = e\mathbb{C}G$ for some idempotent e.

Recommended

Note that recommended problems come from our book. They have answers in the back (I will not grade them, though I am happy to talk about them).

1. Chapter 6: 3, 4

2. Chapter 9: 5

3. Chapter 10: 5, 6