Math 3170: Homework 8

1. Is there a non-connected simple graph on 7 vertices with every vertex at least degree 3?
2. (a) Carefully state what the out-degree and in-degree of a vertex should be in a directed graph.
   (b) State and prove a directed graph version of the theorem on Eulerian trails.
3. (a) Find all nonisomorphic simple graphs on 4 vertices.
   (b) How many nonisomorphic arbitrary graphs are there on four vertices?
4. For which n can one partition the edges of K_n into subsets where each subset is the set of edges of a Hamiltonian path?
5. The n dimensional hypercube Q_n is the simple graph with vertices

$$V = \{(a_1, a_2, \ldots, a_n) \in \{0,1\}^n\},$$

   and an edge between $(a_1, \ldots, a_n)$ and $(b_1, \ldots, b_n)$ if

$$\#\{1 \leq i \leq n \mid a_i = b_i\} = n - 1.$$

   (a) How many vertices does Q_n have?
   (b) What are the degrees of the vertices?
   (c) Why is Q_n called a hypercube?
   (d) Show that for $n \geq 2$, Q_n has a closed Hamiltonian path.