Math 3170: Homework 6

1. (a) Let
$$f_{k,n} = \# \left\{ \begin{array}{c} \text{set partitions of } \{1, 2, \ldots, n\} \\ \text{into } k \text{ subsets that contain} \\ \text{at least 2 elements} \end{array} \right\}.$$

   Find and prove a formula for $f_{k,n}$ in terms of the Stirling numbers of the second kind.

   (b) Let
$$f_n = \# \left\{ \begin{array}{c} \text{set partitions of } \{1, 2, \ldots, n\} \\ \text{into subsets that contain} \\ \text{at least 2 elements} \end{array} \right\}.$$

   Find and prove a formula for f_n in terms of the Bell numbers.

2. Give a self-contained definition of a simple, directed graph.

3. Prove that in a simple graph, if there is a trail between two vertices, then there is also a path between these two vertices.

4. Show that in any simple graph with at least two vertices there are two vertices with the same degree.

   Hint: Focus on a vertex with largest degree, and use the pigeon-hole principle.