

Math 3170: Homework 11

1. The following is true for a party involving n guests.
 - In any group of three guests, there are two that do not know one-another.
 - In any group of seven guests, there are two that know one-another.
 - (a) Give an upper bound on the number of guests based on a Ramsey number (you don't need to know the value of the Ramsey number).
 - (b) Suppose there is a gift exchange and every guest brings a gift for every other guest they know. Show that there are at most $6n$ gifts that come to the party.
Hint: For (b), show that every person knows at most 6 people.
2. Show that if $R(k-1, l)$ and $R(k, l-1)$ are both even, then

$$R(k, l) < R(k-1, l) + R(k, l-1).$$

Hint: Assume that $K_{R(k-1, l) + R(k, l-1) - 1}$ has no red K_k or blue K_l , find the red degree of any vertex, and then count how many red edges it must have.

3. For your project topic, come up with two homework problems:
 - The first should be easy enough that someone can solve it understanding only the basic ideas involved,
 - The second should be harder, solvable only by having read and understood the subtleties of your paper.

Give clear statements and solutions for each.