Math 3170: Homework 8

Due: October 31, 2012

1. Prove that in a simple graph, if there is a trail between two vertices, then there is also a path between these two vertices.

2. Show that in any simple graph with at least two vertices there are two vertices with the same degree.

   Hint: Focus on a vertex with largest degree, and use the pigeon-hole principle.

3. Prove that the number of people who have shaken hands an odd number of times is even.

4. Is there a non-connected simple graph on 7 vertices with every vertex at least degree 3?

5. (a) Carefully state what the out-degree and in-degree of a vertex should be in a directed graph.

   (b) State and prove a directed graph version of the theorem on Eulerian trails.