Math 3170: Homework 6

Due: October 17, 2012

1. (a) For which sequence

$$\begin{aligned} do_n &= \#\{\text{distinct partitions of } n \text{ with odd part sizes}\} \\ sc_n &= \#\{\text{self conjugate partitions of } n\} \end{aligned}$$

   is the ordinary generating function easier to compute?

   (b) Use Homework 5 to find a generating function for both.

2. For $k \in \mathbb{Z}_{\geq 1}$ compute the coefficients a_n in

$$e^{kx} = \sum_{n \geq 0} a_n \frac{x^n}{n!}$$

   in two ways to show that

$$k^n = \sum_{\substack{m_1+m_2+\cdots+m_k=n \\ m_1,m_2,\ldots,m_k \in \mathbb{Z}_{\geq 0}}} \binom{n}{m_1, m_2, \ldots, m_k}.$$

3. Let

$$A(x) = \sum_{n \geq 0} a_n x^n.$$

   (a) Describe the sequence coming from the ordinary generating function

$$\frac{A(x)}{1-x}.$$

   (b) Describe the sequence coming from the exponential generating function

$$\frac{A(x)}{1-x}.$$

4. Give the first 3 terms of the exponential generating function

$$e^{\frac{e^{tx}-1}{t}}.$$

   (The coefficients in your answer should be polynomials in t). These are known as t-Bell numbers.