Math 3170: Homework 5

Due: October 3, 2012

1. Show that the number of partitions of n with even part sizes is the same as the number of partitions of n where each part appears an even number of times.

2. A *self-conjugate* partition is a partition (viewed as a stack of boxes) such that if you reflect across the $y = -x$ axis, you get the same stack of boxes. Let

$$\begin{aligned} do_n &= \#\{\text{distinct partitions of } n \text{ with odd part sizes}\} \\ sc_n &= \#\{\text{self conjugate partitions of } n\} \end{aligned}$$

   Show that for all $n \in \mathbb{Z}_{\geq 0}$, $do_n = sc_n$.

   Hint: Consider in the self-conjugate partition the boxes closest to the walls, and then the boxes 1 box away from the walls, and so on.

3. Let $p_{n,k}$ be the number of integer partitions of n into k parts. Show that

$$p_{n,k} = p_{n-1,k-1} + p_{n-k,k}.$$

4. (a) Let r_n be the number of compositions of n such that each part has size at least 2. Find a recursive formula in terms of r_{n-1} and r_{n-2} for r_n.

   (b) If you replace partitions for compositions in (a), why does your argument cease to work?

   (c) Find a closed formula for r_n.