Math 3170: Homework 1

Due: August 30, 2010

Each problem begins with a more general problem, and concludes with some special cases I'd like you to solve (the numbered problems in bold). You do not need to solve the general case (though you may try to if you'd like).

Derangements

Given n letters and n addressed envelopes, in how many ways can the letters be placed in the envelopes so that no letter is in the correct envelope.

1. Solve the problem for the special cases $n = 3, 4, 5$. Calculate the ratio of this number to $n!$ in each case.

Kirkman's schoolgirls

Fifteen schoolgirls walk each day in five groups of three. Arrange the girls' walks for a week so that, in that time, each pair of girls walks together in a group just once.

2. Solve this problem for nine schoolgirls walking for four days.

Knight's Tour Problem

Let K_{mn} be an $m \times n$ chess-board. Is there a way to move a knight around the board so that he lands on every square exactly once?

3. Solve this problem for 3×3, 3×4, and 3×5 chess-boards. Are there any more general statements you can make (find at least one infinite family of chessboards for which no knight's tour exists)?

A Ramsey game

This two-player game requires a sheet of paper and pencils of two colours, say red and blue. Six points on the paper are chosen, with no three in line. Now the players take a pencil each, and take turns drawing a line connecting two of the chosen points. The first player to complete a triangle of his/her own colour loses. (Only triangles with vertices at the chosen points count). Can the game ever result in a draw?

4. Test the assertion that the Ramsey game cannot end in a draw by playing it with a friend. Can you prove it? Develop heuristic rules for successful play.