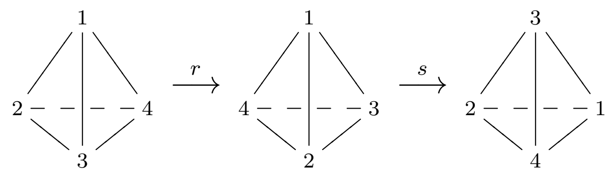
Math 3140: Homework 1

Due: Wednesday, September 3

A. Figure 1.4 essentially describes two symmetries of the tetrahedron:



- 1.3 Adopt the notation of Figure 1.4. Show that the axis of the composite rotation srs passes through the vertex 4, and that the axis of $rsrr$ is determined by the midpoints of edges 12 and 34.
- 1.4 Having completed 1.3, Express each of the twelve rotational symmetries of the tetrahedron in terms of r and s.
- 1.5 Again with the notation of Figure 1.4, check that $r^{-1} = rr$, $s^{-1} = s$, $(rs)^{-1} = srr$, and $(sr)^{-1} = rrs$.
- 1.9 Find all plane symmetries (rotations and reflections) of a regular pentagon.

B. 2.5 A function from the plane to itself which preserves the distance between any two points is called and *isometry*. Prove that an isometry must be a bijection and check that the collection of all isometries of the plane forms a group under composition of functions.

- 2.6 Show that the collection of all rotations of the plane about a fixed point P forms a group under composition of functions. Is the same true of the set of all reflections in lines which pass through P? What happens if we take all the rotations and all the reflections?
- 2.7 Let x and y be elements of a groups G. Prove that G contains elements w, z which satisfy $wx = y$ and $xz = y$, and show that these elements are unique.
- 2.8 If x and y are elements of a group, prove that $(xy)^{-1} = y^{-1}x^{-1}$.

C. What are all the group structures on the set $\{1, a, b, c\}$, where you may assume 1 is the identity?