D_4	1	r	r^2	r^3	s	rs	r^2s	r^3s
1	1	r	r^2	r^3	s	rs	r^2s	r^3s
$oldsymbol{r}$	r	r^2	r^3	1	rs	r^2s	r^3s	s
r^2	r^2	r^3	1	r	r^2s	r^3s	s	rs
r^3	r^3	1	r	r^2	r^3s	S	rs	r^2s
s	s	r^3s	r^2s	rs	1	r^3	r^2	r
rs	rs	s	r^3s	r^2s	r	1	r^3	r^2
r^2s	r^2s	rs	s	r^3s	r^2	r	1	r^3
r^3s	r^3s	r^2s	rs	s	r^3	r^2	r	1

D_4	1	r	r^2	r^3	s	rs	r^2s	r^3s
1	1	r	r^2	r^3	s	rs	r^2s	r^3s
$oldsymbol{r}$	$oldsymbol{r}$	r^2	r^3	1	rs	r^2s	r^3s	s
r^2	r^2	r^3	1	r	r^2s	r^3s	S	rs
r^3	r^3	1	r	r^2	r^3s	s	rs	r^2s
S	s	r^3s	r^2s	rs	1	r^3	r^2	r
rs	rs	\boldsymbol{s}	r^3s	r^2s	r	1	r^3	r^2
r^2s	r^2s	rs	s	r^3s	r^2	r	1	r^3
r^3s	r^3s	r^2s	rs	s	r^3	r^2	r	1

$$H = \{1, r, r^2, r^3\}$$

D_4	1	r	r^2	r^3	s	rs	r^2s	r^3s
1	1	r	r^2	r^3	s	rs	r^2s	r^3s
$oldsymbol{r}$	$oldsymbol{r}$	r^2	r^3	1	rs	r^2s	r^3s	$oldsymbol{s}$
r^2	r^2	r^3	1	$oldsymbol{r}$	r^2s	r^3s	s	rs
r^3	r^3	1	r	r^2	r^3s	$oldsymbol{s}$	rs	r^2s
s	s	r^3s	r^2s	rs	1	r^3	r^2	$oldsymbol{r}$
rs	rs	\boldsymbol{s}	r^3s	r^2s	r	1	r^3	r^2
r^2s	r^2s	rs	s	r^3s	r^2	r	1	r^3
r^3s	r^3s	r^2s	rs	s	r^3	r^2	r	1

$$H = \{1, r, r^2, r^3\}$$

D_4	1	r	r^2	r^3	s	$\mid rs \mid$	r^2s	$\left \ r^3 s \ \right $
1	1	r	r^2	r^3	s	rs	r^2s	r^3s
r	r	r^2	r^3	1	rs	r^2s	r^3s	s
r^2	r^2	r^3	1	r	r^2s	r^3s	s	rs
r^3	r^3	1	r	r^2	r^3s	s	rs	r^2s
s	s	r^3s	r^2s	rs	1	r^3	r^2	r
rs	rs	s	r^3s	r^2s	r	1	r^3	r^2
r^2s	r^2s	rs	s	r^3s	r^2	r	1	r^3
r^3s	r^3s	r^2s	rs	s	r^3	r^2	r	1

$$H = \{1, r, r^2, r^3\}$$

D_4	1	r^2	r	r^3	s	r^2s	$\mid rs \mid$	r^3s
1	1	r^2	r	r^3	s	r^2s	rs	r^3s
r^2	r^2	1	r^3	r	r^2s	s	r^3s	rs
r	r	r^3	r^2	1	rs	r^3s	r^2s	s
r^3	r^3	r	1	r^2	r^3s	rs	s	r^2s
S	s	r^2s	r^3s	rs	1	r^2	r^3	r
r^2s	r^2s	s	rs	r^3s	r^2	1	r	r^3
rs	rs	r^3s	\boldsymbol{s}	r^2s	r	r^3	1	r^2
r^3s	r^3s	rs	r^2s	$oldsymbol{s}$	r^3	r	r^2	1

$$H = \{1, r^2\}$$

D_4	1	r^2	r	r^3	s	r^2s	$\mid rs \mid$	$\int r^3 s$
1	1	r^2	r	r^3	s	r^2s	rs	r^3s
r^2	r^2	1	r^3	r	r^2s	s	r^3s	rs
$oldsymbol{r}$	r	r^3	r^2	1	rs	r^3s	r^2s	s
r^3	r^3	r	1	r^2	r^3s	rs	s	r^2s
$oldsymbol{s}$	s	r^2s	r^3s	rs	1	r^2	r^3	r
r^2s	r^2s	s	rs	r^3s	r^2	1	r	r^3
rs	rs	r^3s	\boldsymbol{s}	r^2s	r	r^3	1	r^2
r^3s	r^3s	rs	r^2s	$oldsymbol{s}$	r^3	r	r^2	1

$$H=\{1,r^2\}$$

D_4	1	r^2	r	r^3	s	r^2s	$\mid rs \mid$	r^3s
1	1	r^2	r	r^3	s	r^2s	rs	r^3s
r^2	r^2	1	r^3	r	r^2s	s	r^3s	rs
r	$oldsymbol{r}$	r^3	r^2	1	rs	r^3s	r^2s	s
r^3	r^3	r	1	r^2	r^3s	rs	s	r^2s
$oldsymbol{s}$	s	r^2s	r^3s	rs	1	r^2	r^3	r
r^2s	r^2s	s	rs	r^3s	r^2	1	r	r^3
rs	rs	r^3s	s	r^2s	r	r^3	1	r^2
r^3s	r^3s	rs	r^2s	s	r^3	r	r^2	1

$$H = \{1, r^2\}$$

D_4	1	$oldsymbol{s}$	r^2	r^2s	r	rs	r^3	r^3s
1	1	\boldsymbol{s}	r^2	r^2s	r	rs	r^3	r^3s
s	s	1	r^2s	r^2	r^3s	r^3	rs	r
r^2	r^2	r^2s	1	s	r^3	r^3s	r	rs
r^2s	r^2s	r^2	s	1	rs	r	r^3s	r^3
r	r	rs	r^3	r^3s	r^2	r^2s	1	s
rs	rs	r	r^3s	r^3	S	1	r^2s	r^2
r^3	r^3	r^3s	r	rs	1	s	r^2	r^2s
r^3s	r^3s	r^3	rs	r	r^2s	r^2	\boldsymbol{s}	1

$$H=\{1,s\}$$

D_4	1	$oldsymbol{s}$	r^2	r^2s	r	$\mid rs \mid$	r^3	$\int r^3 s$
1	1	\boldsymbol{s}	r^2	r^2s	r	rs	r^3	r^3s
$oldsymbol{s}$	\boldsymbol{s}	1	r^2s	r^2	r^3s	r^3	rs	r
r^2	r^2	r^2s	1	s	r^3	r^3s	r	rs
r^2s	r^2s	r^2	s	1	rs	r	r^3s	r^3
$oldsymbol{r}$	r	rs	r^3	r^3s	r^2	r^2s	1	s
rs	rs	r	r^3s	r^3	S	1	r^2s	r^2
r^3	r^3	r^3s	r	rs	1	s	r^2	r^2s
r^3s	r^3s	r^3	rs	r	r^2s	r^2	\boldsymbol{s}	1

$$H=\{1,s\}$$

D_4	1	$oldsymbol{s}$	r^2	r^2s	r	$\mid rs \mid$	r^3	$\int r^3 s$
1	1	s	r^2	r^2s	r	rs	r^3	r^3s
$oldsymbol{s}$	S	1	r^2s	r^2	r^3s	r^3	rs	r
r^2	r^2	r^2s	1	s	r^3	r^3s	r	rs
r^2s	r^2s	r^2	s	1	rs	r	r^3s	r^3
$oldsymbol{r}$	$m{r}$	rs	r^3	r^3s	r^2	r^2s	1	s
rs	rs	r	r^3s	r^3	s	1	r^2s	r^2
r^3	r^3	r^3s	r	rs	1	s	r^2	r^2s
r^3s	r^3s	r^3	rs	r	r^2s	r^2	s	1

$$H=\{1,s\}$$