Math 3140: Homework 11

Due: Wednesday, November 20

17.1. Let $G = \langle (1,2,3)(4,5), (7,8) \rangle \subseteq S_8$. Then G acts on the set $X = \{1,2,3,4,5,6,7,8\}$. Calculate the orbits of the G-action in X, and the stabilizer of each element.

17.4 Let G act on a set X, and let O be an orbit in G. Show that $\mathrm{Stab}_G(x) = \mathrm{Stab}_G(y)$ for all $x, y \in O$ if and only if $\mathrm{Stab}_G(x) \lhd G$.

17.7-8. **The diagonal action.** Suppose G acts on X and Y. Let

$$\begin{array}{ccc} G \times (X \times Y) & \longrightarrow & X \times Y \\ (g, (x, y)) & \mapsto & (g(x), g(y)) \end{array} \qquad (*)$$

(a) Show that $(*)$ gives an action of G on $X \times Y$.

(b) Find the stabilizer of $(x, y) \in X \times Y$.

(c) Give an example to show that this action is not necessarily transitive, even if G acts transitively on both X and Y.

(d) Find the orbits and stabilizers of $G = \langle (1,2,3,4), (2,4) \rangle \subseteq S_4$ acting on $X \times X$ diagonally, where $X = \{1,2,3,4\}$.

17.10. The *centralizer* of $g \in G$ is the subgroup

$$C_G(g) = \{h \in G \mid hgh^{-1} = g\}.$$

(a) Identify a set X with an action of G on that set such that $C_G(g) = \mathrm{Stab}_G(g)$ for that action.

(b) Show that if some conjugacy class of G has exactly two elements, then G is not simple.