Math 3140: Homework 7

Due: Wednesday, October 19

A. A *ring* R is a set with two functions

$$\begin{aligned} \cdot : \quad R \times R &\longrightarrow R \\ (x, y) &\mapsto x \cdot y \end{aligned} \qquad \text{and} \qquad \begin{aligned} + : \quad R \times R &\longrightarrow R \\ (x, y) &\mapsto x + y \end{aligned}$$

such that

(1) $(R, +)$ is an abelian group,

(2) For all $a, b, c \in R$,
- $a \cdot (b + c) = a \cdot b + a \cdot c$,
- $(a + b) \cdot c = a \cdot c + b \cdot c$,
- $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Do the following

(a) Show that $M_n(\mathbb{R})$ is a ring.

(b) Give a definition for what you think a subring should be.

(c) Give a definition for what you think a ring isomorphism should be.

B. 11.4 Suppose $|G|$ is the product of two distinct primes. Show that any proper subgroup of G must be cyclic.

11.7 Suppose $n \in \mathbb{Z}_{\geq 1}$ and m divides $2n$. Show that D_n contains a group of order m.

11.8 Does A_5 contain a subgroup of order m for every m that divides $|A_5| = 60$?

C.

12.4-5 Find examples of a group G and a subgroup H such that the following sets are **not** equivalence relations:

(a) $\{(x, y) \mid xy \in H\}$,

(b) $\{(x, y) \mid xyx^{-1}y^{-1} \in H\}$.

12.8 Let H be a subgroup of a group G.

(a) Show that if $|G| = 2|H|$, then $gH = Hg$ for all $g \in G$.

(b) Show that $gH = Hg$ for all $g \in G$ if and only if $ghg^{-1} \in H$ for all $h \in H$, $g \in G$.