

# Math 3140: Homework 3

Due: Wednesday, September 14

A. 5.4, 5.10

B. For  $D_n$ , let the vertices of the regular  $n$ -gon be labeled by  $1, 2, \dots, n$  in a clockwise direction. Let  $s$  be symmetry that sends the vertex 1 to  $n$  and the vertex 2 to  $n - 1$ . Let  $t$  be the symmetry that sends 1 to 1 and sends 2 to  $n$ . Show that

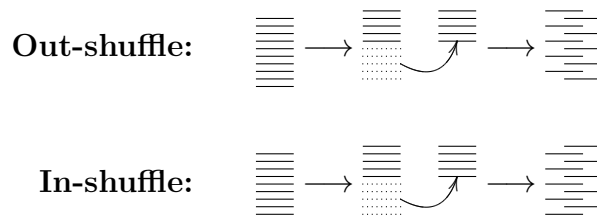
$$D_n = \langle s, t \rangle.$$

C. Find examples of the following (justify your answer), or explain why they don't exist:

- (1) A nonabelian cyclic group.
- (2) A group generated by two elements.
- (3) A group generated by three elements, but not by two element.

D. Show that  $(\mathbb{Q}, +)$  is not cyclic. In fact, show that  $(\mathbb{Q}, +)$  does not even have a finite number of generators. Hint: What does the subgroup  $\langle p/q \rangle$  look like for  $p, q \in \mathbb{Z}_{\neq 0}$ ?

E. A *perfect shuffle* is a shuffle where you split the deck into two equal stacks (the top stack and the bottom stack) and push the stacks together in such a way that the cards from the two stacks alternate. There are two ways of doing this: either the bottom card came from the top stack (an *in-shuffle*), or the bottom card came from the bottom stack (an *out-shuffle*). For example, if you have 10 cards,



By a judicious use of in and out-shuffles one can create all kinds of card tricks (providing one is capable of a perfect shuffle), since such shuffling is in no way random. Suppose you have a deck of 52 cards.

- (1) Prove that if you apply the in-shuffle enough times, you will get back with what you started. Hint: prove that you get back where you started without actually doing it (it's a lot of shuffles).
- (2) How many out-shuffles do you need to get back where you started (Justify your answer)?

Remark: It turns out with normal shuffling (which is far from perfect) it takes approximately 7 shuffles to get close to random (which is proved in part using methods of this class).