Math 3140: Homework 10

Due: Wednesday, November 9

A.

15.13. Suppose H is a cyclic normal subgroup of G. Show that any subgroup of H is also normal in G.

(2) A group G is *meta-abelian* if there exists and abelian normal subgroup $A \lhd G$ such that G/A is also abelian. Show that G is meta-abelian if and only if $[[G, G], [G, G]] = 1$ (the commutator subgroup of the commutator subgroup).

(3) Find the commutator subgroup of $W_{2,n}$.

B. 16.4. Show that if $A \lhd G$ and $B \lhd H$, then $(A \times B) \lhd (G \times H)$ and

$$(G \times H)/(A \times B) \cong (G/A) \times (G/B).$$

16.11 (Fourth Isomorphism Theorem) Let $\varphi : G \to H$ be a surjective homomorphism with kernel K.

(a) For every subgroup $B \subseteq H$, show that the set

$$\varphi^{-1}(B) = \{g \in G \mid \varphi(g) \in B\}$$

is a subgroup of G that contains K.

(b) Show that there is a bijection between

$$\left\{ \begin{array}{c} \text{Subgroups } A \subseteq G \\ \text{such that } K \subseteq A \end{array} \right\} \longleftrightarrow \left\{ \text{ Subgroups } B \subseteq H \right\}.$$

16.12 An *maximal* normal subgroup N of G is a normal subgroup such that if $H \supseteq N$ is a normal subgroup of G, then $H = N$ or $H = G$. Show that N is a maximal normal subgroup of G if and only if G/N is simple.