

Project 1

Due: October 12, 2007

Recommended length: 2-3 pages.

Format: typed.

The goal of this assignment is to examine the behavior of in and out shuffles. Suppose that you have a deck of $2n$ cards. Let $i \in S_{2n}$ denote the permutation that gives an in-shuffle, and let $o \in S_{2n}$ denote the permutation that gives the out-shuffle. Let

$$B_n = \left\{ \begin{array}{l} n \times n \text{ matrices with entries in } \{-1, 0, 1\} \\ \text{and exactly one nonzero entry in} \\ \text{every row and in every column} \end{array} \right\}$$

The goal of the project is to prove the following theorem.

Theorem. *The group $\langle i, o \rangle$ is isomorphic to a subgroup of B_n .*

In doing this you should

- (a) Briefly introduce the topic.
- (b) Give the necessary definitions and results you will need for the theorem. You do not need to prove the results that are in the book, but both the results and the definitions should be stated in your own words in a way that focuses them on the theorem. You may assume that the reader has read up through Chapter 8 of the textbook.
- (c) State and prove the theorem.
- (d) Give an example of a permutation in S_{2n} which is not in $\langle i, o \rangle$.

Suggestions for the proof:

- (1) Show that any permutation that is in $\langle i, o \rangle$ is *horizontally symmetric* (explain what that means),
- (2) Show that the subgroup of horizontally symmetric permutations is isomorphic to B_n .
- (3) Draw the appropriate conclusions.

Note that this is a writing assignment, so the main focus should be on clearly communicating the ideas in the proof. I recommend looking at your favorite mathematics texts and trying to emulate their style. I also suggest you have another member of the class read through a draft before handing it in.