

Math 3140: Homework 8

Due: Wednesday, October 31

A. (1) Find an example groups G , H and a homomorphism $\varphi : G \rightarrow H$ such that φ is neither surjective nor injective.

14.5 Prove that the 3-cycles of A_5 form a single conjugacy class. Find two 5-cycles which are *not* conjugate in A_5 (though they are conjugate in S_5).

(3) Let B_n be as in Project 1.

(a) Classify the conjugacy classes of B_2 in a way analogous to our cycle-type classification of permutations.

(b) Classify the conjugacy classes of B_n for all n . That is, develop a general analogue to “cycle type” for signed permutations.

Hint: You will be stacking boxes into two corners.

B. 15.2 Find all normal subgroups of D_n .

15.7 Let $K \triangleleft G \times H$ be such that

$$K \cap (\{1_G\} \times H) = \{(1_G, 1_H)\} = K \cap (G \times \{1_H\}).$$

Show that K must be abelian.

15.12 Find a proper normal subgroup of A_4 . Show that any non-trivial normal subgroup H of A_5 must contain a 3-cycle, and use 14.5 to conclude that $H = A_5$, thereby proving A_5 is simple.