

Math 3140: Homework 7

Due: Wednesday, October 24

A.

12.4-5 Find examples of a group G and a subgroup H such that the following sets are **not** equivalence relations:

- (a) $\{(x, y) \mid xy \in H\}$,
- (b) $\{(x, y) \mid xyx^{-1}y^{-1} \in H\}$.

12.8 Let H be a subgroup of a group G .

- (a) Show that if $|G| = 2|H|$, then $gH = Hg$ for all $g \in G$.
- (b) Show that $gH = Hg$ for all $g \in G$ if and only if $ghg^{-1} \in H$ for all $h \in H$, $g \in G$.

B. 14.2 Find the conjugacy classes for D_n for all n (be careful to distinguish between different cases).

14.3 Suppose $\varphi : G \rightarrow H$ is an isomorphism of groups, and suppose C is a conjugacy class of G . Show that $\varphi(C)$ is a conjugacy class of H .

- (3) Show that if $w \in S_n$, then both w and w^{-1} are in the same conjugacy class. Find an example of a group for which this is not true.

14.10 Find the center of D_n for all n .

- (5) Suppose G is a matrix group. Show that if $g, h \in G$ are in the same conjugacy class, then $\det(g) = \det(h)$.