Math 3140: Homework 6

Due: Wednesday, October 17

A. 9.1 Do either of the following sets of $n \times n$ matrices form a group?

   (a) Diagonal matrices, $\{a \in M_n(\mathbb{R}) \mid a_{ij} = 0, i \neq j, a_{ii} \neq 0\}$.

   (b) Symmetric matrices, $\{a \in M_n(\mathbb{R}) \mid a_{ij} = a_{ji}, 1 \leq i, j \leq n\}$.

(2) Let $C_r = \langle x \rangle$ be the cyclic group with r elements (but written with multiplication, rather than addition). Let

$$W_{r,n} = \left\{ a \in M_n(C_r \cup \{0\}) \;\middle|\; \begin{array}{c} a \text{ has exactly one nonzero entry} \\ \text{in every row and every column} \end{array} \right\}.$$

   (a) Show that $W_{r,n}$ is a group. What groups are $W_{1,n}$ and $W_{2,n}$ isomorphic to?

   (b) What is the order of $W_{r,n}$? Show that $W_{2,2} \cong D_4$.

B. 11.4 Suppose $|G|$ is the product of two distinct primes. Show that any proper subgroup of G must be cyclic.

11.7 Suppose $n \in \mathbb{Z}_{\geq 1}$ and m divides $2n$. Show that D_n contains a group of order m.

11.8 Does A_5 contain a subgroup of order m for every m that divides $|A_5| = 60$?