Math 3140: Homework 3

Due: Wednesday, September 19

A. 6.3. Show that the elements $w \in S_9$ such that $\{w(2), w(5), w(7)\} = \{2, 5, 7\}$ form a subgroup of $S_9$. What is the order of this subgroup?

6.7+. (a) Describe/characterize the elements of order 2 of $S_n$.

(b) Show that if $n \geq 4$, then every permutation can be written as a product of two permutations of order 2. Hint: Answer the question first for cyclic permutations.

(c) What goes wrong if $n < 4$?

B. 7.5. Let $G$ be a group. Show that the function

$$\varphi : G \rightarrow G : x \mapsto x^{-1}$$

is an isomorphism if and only if $G$ is abelian.

7.9. Suppose $G$ is cyclic with generator $x \in G$. Show that if $\varphi : G \rightarrow H$ is an isomorphism, then $\varphi$ is completely determined by $\varphi(x)$. Show that $H = \langle \varphi(x) \rangle$.

C. (1) Show that there are exactly two groups with four elements (up to isomorphism).

(2) The braid group $B_n$ is a group generated by the diagrams of $S_n$ but we keep track of where strings cross. For example,

becomes

or

and we keep track of these crossings when multiplying,

What is the inverse of an element in $B_n$? Show that $B_n$ has infinite order. What are the elements of finite order in $B_n$?