Math 3140: Homework 12

Due: Wednesday, December 5

20.1 (a) Show that if $|G| = 126$, then G has a nontrivial proper normal subgroup.

(b) Show that if $|G| = 1000$, then G is not simple.

(c) Suppose $|G| = p^k m$ where p is prime and p does not divide m. Prove that if $p > m$, then G is not simple.

(1) Prove that if G is abelian and simple, then $G \cong \mathbb{Z}_p$ for some prime number p.

20.3. (a) Prove that if all the Sylow subgroups are normal, then G is isomorphic to the direct product of of its Sylow subgroups.

(b) If you know that G is abelian, and $|G| = 154000$, then what do you know about G?

20.7. Classify the groups of order $p^2 q$ if p is not congruent to ± 1 modulo q (and $p \neq q$ are prime).