Math 3140: Homework 12

Due: Wednesday, December 5

20.1 (a) Show that if $|G| = 126$, then $G$ has a nontrivial proper normal subgroup.
(b) Show that if $|G| = 1000$, then $G$ is not simple.
(c) Suppose $|G| = p^k m$ where $p$ is prime and $p$ does not divide $m$. Prove that if $p > m$, then $G$ is not simple.

(1) Prove that if $G$ is abelian and simple, then $G \cong \mathbb{Z}_p$ for some prime number $p$.

20.3. (a) Prove that if all the Sylow subgroups are normal, then $G$ is isomorphic to the direct product of its Sylow subgroups.
(b) If you know that $G$ is abelian, and $|G| = 154000$, then what do you know about $G$?

20.7. Classify the groups of order $p^2 q$ if $p$ is not congruent to $\pm 1$ modulo $q$ (and $p \neq q$ are prime).