Math 3140: Homework 11

Due: Wednesday, December 5

A. 17.1. Let $G = \langle (1, 2, 3)(4, 5), (7, 8) \rangle \subseteq S_8$. Then $G$ acts on the set $X = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Calculate the orbits of the $G$-action in $X$, and the stabilizer of each element.

17.7-8. The diagonal action. Suppose $G$ acts on $X$ and $Y$. Let

$$G \times (X \times Y) \to X \times Y$$

$$ (g, (x, y)) \mapsto (g(x), g(y))$$

i. Show that (*) gives an action of $G$ on $X \times Y$.

ii. Find the stabilizer of $(x, y) \in X \times Y$.

iii. Give an example to show that this action is not necessarily transitive, even if $G$ acts transitively on both $X$ and $Y$.

iv. Find the orbits and stabilizers of $G = \langle (1, 2, 3, 4), (2, 4) \rangle \subseteq S_4$ acting on $X \times X$ diagonally, where $X = \{1, 2, 3, 4\}$.

17.10. Recall that if $x \in G$, then the stabilizer of $G$ in $X$ is

$$C_G(x) = \{ g \in G \mid gxg^{-1} = x \}.$$ 

(This is the stabilizer of an element under the conjugation action of $G$ on $X = G$). Show that if some conjugacy class of $G$ has exactly two elements, then $G$ is not simple.

B. 18.1 Use orbit counting methods to find the number of distinct ways to paint the edges of a cube with two colors.

18.7. How many different ways are there of coloring the vertices and edges of a regular hexagon using red, black or yellow for the edges and black or white for the vertices?