Math 2001: WHW2

Due February 24, 2016

The following assignment will be graded with an emphasis on clarity of exposition. You should write in complete sentences, be as precise as possible, and be mathematically correct (both in content and format). Be sure to include a title, a short introduction to the topic, and to define all the relevant mathematical terms. The assignment should be **typed**, and emailed as a .pdf to me by 2pm on the day it is due.

Explain why the number of shortest paths in an $m \times n$ lattice from one corner to another is

$$\binom{s}{r}$$

where

$$s = \begin{array}{c} \text{total \# of} \\ \text{steps} \end{array} \quad \text{and} \quad r = \begin{array}{c} \text{total \# of} \\ \text{right steps} \end{array}$$

You should include

(a) An introduction

(b) Set up the problem

(c) What do you mean "shortest path?" Ie. What is s in terms of m and n?

(d) A precise statement of your main theorem.

(e) To prove your theorem, you should carefully describe three things:

- Given a shortest path p, how does this determine some r-element subset of an s-element set A_p?

- Given an r-element subset B of an s-element set, how does this give a shortest path q_B?

- Show that for your recipes $q_{A_p} = p$ (that is, take a path p and create a set A_p, then create a path again q_{A_p}), and $A_{q_B} = B$ (that is, take a set B and create a path q_B, then create a set again A_{q_B}).

Examples are helpful, but do not constitute proof.

Extra Credit (5 points)

Type your paper in LaTeX. If you do this, send both a .pdf and the .tex source file.