Math 2001: PHW9

Due: March 30, 2016

1. Consider the sequences given by

$$a_n = 3a_{n-1} - 2a_{n-2},$$

   with $a_1 = 3, a_2 = 7$. Show that $a_n = 2^{n+1} - 1$.

2. For $n \in \mathbb{Z}_{\geq 1}$, consider the sets

$$E_n = \{\text{binary sequences of length } n \text{ with an even number of 1's}\}$$
$$O_n = \{\text{binary sequences of length } n \text{ with an odd number of 1's}\}$$

   (a) Use a counting argument to show that $|E_n| = |O_n|$.

   (b) Show that $|E_1|, |E_2|, \ldots$ is a recursive sequence, and give a counting argument for the recursion.

3. Let h_n be the number of ways to color the squares of an $1 \times n$ chessboard with red and blue so that no two red squares are adjacent. Find a recurrence relation for h_n.