Math 2001: PHW11

1. From the book do:

   12.1. 4, 8

   12.2. 4, 10, 14

   12.4. 4, 10

2. Let R_n be the set of ways to place n non-attacking rooks on an $n \times n$ chess-board.

   (a) Let $f : R_n \to \mathbb{Z}$ be given by

   $$f(r) = \text{number of rooks on the diagonal squares of } r, \qquad \text{for } r \in R_n.$$

   For example, if $n = 4$,

   $$f\left(\begin{array}{|c|c|c|c|}\hline ⊓ & & & \\\hline & & & ⊓ \\\hline & & ⊓ & \\\hline & ⊓ & & \\\hline\end{array}\right) = 2, \qquad \text{where} \quad \begin{array}{|c|c|c|c|}\hline ⊓ & & & \\\hline & & & ⊓ \\\hline & & ⊓ & \\\hline & ⊓ & & \\\hline\end{array} \in R_4,$$

   and I've shaded the diagonal squares.

      i. What is $f(R_5)$?

     ii. Is f injective?

    iii. Is f surjective?

   (b) Find an injective function $g : R_n \to \mathbb{Z}$ (without changing the sets R_n and $\mathbb{Z}$).