

Math 2001: PHW11

1. From the book do:

11.4. 4, 6

12.1. 4, 8

12.2. 4, 10, 14

2. Let R_n be the set of ways to place n non-attacking rooks on an $n \times n$ chess-board.

(a) Let $f : R_n \rightarrow \mathbb{Z}$ be given by

$$f(r) = \text{number of rooks on the diagonal squares of } r, \quad \text{for } r \in R_n.$$

For example, if $n = 4$,

$$f \left(\begin{array}{|c|c|c|c|} \hline \text{R} & & & \\ \hline & & & \text{R} \\ \hline & & \text{R} & \\ \hline & \text{R} & & \\ \hline \end{array} \right) = 2, \quad \text{where} \quad \begin{array}{|c|c|c|c|} \hline \text{R} & & & \\ \hline & & & \text{R} \\ \hline & & \text{R} & \\ \hline & \text{R} & & \\ \hline \end{array} \in R_4,$$

and I've shaded the diagonal squares.

- i. What is $f(R_n)$?
 - ii. Is f injective?
 - iii. Is f surjective?
- (b) Find an injective function $g : R_n \rightarrow \mathbb{Z}$ (without changing the sets R_n and \mathbb{Z}).