

Math 2001: PHW8

1. Use induction to prove the statements

- (a) $1 + 2 + \cdots + n = \binom{n+1}{2}$,
- (b) $(1 + 2 + \cdots + n)^2 = 1^3 + 2^3 + \cdots + n^3$.

2. For each of the following sequences,

- Give a formula for the n th term in the sequence,
- Give a recursive definition for the sequence (ie. initial values and a recursive equation).

- (a) $\{1, 2, 3, 4, 5, \dots\}$
- (b) $\{1, 2, 4, 8, 16, 32, \dots\}$
- (c) $\{1, 2, 6, 24, 120, \dots\}$

3. Let f_0, f_1, \dots be the Fibonacci sequence. For each of the following

- Decide whether the identity is easier to prove by induction or directly using Binet's formula (and some algebra). Explain.
- Prove the identity using your preferred method.

(a) $\sum_{k=1}^n f_k^2 = f_n f_{n+1}$.

(b) $\sum_{k=0}^n f_k = f_{n+2} - 1$.

(c) $f_{2n+1} = f_{n+1}^2 + f_n^2$.

(d) $f_{2n} = f_{n+1}^2 - f_{n-1}^2$.

4. The Lucas sequence is given by

$$L_1 = 1, \quad L_2 = 3, \quad L_n = L_{n-1} + L_{n-2}, \quad n \geq 3.$$

- (a) Find the first 6 values of the Lucas sequence.
- (b) What should L_0 be defined to be to not mess up the recursion?
- (c) Use induction to prove that

$$L_n = f_{n-1} + f_{n+1}, \quad \text{for } n \geq 1,$$

where f_n is the n th Fibonacci number.

(d) Prove that

$$L_n = \left(\frac{1 + \sqrt{5}}{2} \right)^n + \left(\frac{1 - \sqrt{5}}{2} \right)^n.$$