Math 2001: PHW 1

1. If we assume that

   "There is always sun when the wind is in the East,"

   then which of the following statements are true:

   (a) If it's sunny, then the wind must be in the East,
   (b) If the wind isn't in the East, then it must be overcast,
   (c) If it's overcast, then the wind must not be in the East.

   Give an explanation of your answers.

2. Consider the two statements

   - "For each BLAH there exists BLOOF."
   - "For all BLAH there exists BLOOF."

   Either explain why they are the same, or give example BLAHs and BLOOFs to show how they are different.

3. Consider the statement

   Theorem. *Suppose G is a connected graph. If G has an Eulerian circuit, then every vertex of G has even degree.*

   Without necessarily understanding what this says, answer the following.

   (a) What are the assumptions of the statement (if any)?
   (b) What are the conclusions of the statement (if any)?

4. Analyze tic-tac-toe.

   (a) Is there always a winner?
   (b) What is an optimal strategy?
   (c) Does the optimal strategy depend on whether you go first or second?

   Justify your answers as well as possible.

5. How many ways are there of placing 4 non-attacking rooks on an 4×4 chess-board? What about 5 non-attacking rooks on a 5×5 chessboard. Example: there are 2 ways of placing 2 non-attacking rooks on a 2×2 chessboard,
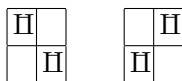
   Can you formulate a guess for how many ways there are to place n non-attacking rooks on an $n \times n$ chessboard?

6. There's a party with 7 guests. If each guest shakes the hand of every other guest, how many handshakes are there?