Math 2001: Homework P5

Due: October 2, 2013

1. From the book do problems:

    (a) 2.2.4, 2.2.6, 2.2.11.

    (b) 3.1.6, 3.1.10, 3.1.20 (Hint: Use induction and the fact that $13^{k+1} = 13(13^k - 1 + 1)$.)

2. Consider the following

   Claim. *The number $n(n+1)$ is an odd number for every n.*

   Proof. Assume the statement is true for n. We prove the statement for $n+1$ by induction. Note that
   $$(n+1)((n+1)+1) = n(n+1) + 2(n+1).$$
   By induction $n(n+1)$ is odd. Thus, $(n+1)((n+1)+1)$ is the sum of an odd number $n(n+1)$ and an even number $2(n+1)$. The sum of an odd number and an even number is odd. Thus, we have proved the claim by induction. $\square$

   I checked the claim and it doesn't seem to work for $n = 15$, since $15 \cdot 16 = 240$, which is even. What is wrong with the proof?

3. Consider the set
   $$A = \{4n + 1 \in \mathbb{Z} \mid n \in \mathbb{Z}, n \geq 0\}.$$

    (a) Describe the set A in words.

    (b) Show that the product of any two elements in A is another element in A.