

## Math 2001: Homework W3

Due October 25, 2013

The following assignment will be graded with an emphasis on clarity of exposition. You should write in complete sentences, be as precise as possible, and be mathematically correct (both in content and format). Be sure to include a title, a short introduction to the topic, and to define all the relevant mathematical terms. The assignment should be **typed**, and emailed as a .pdf to me by 2pm on the day it is due.

Explain why the number of shortest paths in an  $m \times n$  lattice from one corner to another is

$$\binom{s}{r}$$

where

$$s = \begin{array}{l} \text{total \# of} \\ \text{steps} \end{array} \quad \text{and} \quad r = \begin{array}{l} \text{total \# of} \\ \text{right steps} \end{array}$$

You should include

- (a) An introduction
- (b) Set up the problem
- (c) What do you mean “shortest path?” I.e. What is  $s$  in terms of  $m$  and  $n$ ?
- (d) A precise statement of your main theorem.
- (e) To prove your theorem, you should carefully describe three things:
  - Given a shortest path  $p$ , how does this determine some  $r$ -element subset of an  $s$ -element set  $A_p$ ?
  - Given an  $r$ -element subset  $B$  of an  $s$ -element set, how does this give a shortest path  $q_B$ ?
  - Show that for your recipes  $q_{A_p} = p$  (that is, take a path  $p$  and create a set  $A_p$ , then create a path again  $q_{A_p}$ ), and  $A_{q_B} = B$  (that is, take a set  $B$  and create a path  $q_B$ , then create a set again  $A_{q_B}$ ).

Examples are helpful, but do not constitute proof. The beginning of Chapter 4.1 may be helpful, though it doesn't completely do the problem for you.