Math 2001: Homework P12

Due: December 11, 2013

1. From the book, do problems:

   (a) 6.2: 1, 3 (a–e), 6(a–d)

2. For each of the following sequences,

   - Give a formula for the nth term in the sequence,
   - Give a recursive definition for the sequence (ie. initial values and a recursive equation).

   (a) $\{1, 2, 3, 4, 5, \ldots\}$

   (b) $\{1, 2, 4, 8, 16, 25, \ldots\}$

   (c) $\{1, 2, 6, 24, 120, \ldots\}$

3. Let $f_0, f_1, \ldots$ be the Fibonacci sequence. For each of the following

   - Decide whether the identity is easier to prove by induction or directly using Binet's formula (and some algebra). Explain.
   - Prove the identity using your preferred method.

   (a) $\sum_{k=0}^{n} f_k = f_{n+2} - 1.$

   (b) $f_{2n+1} = f_{n+1}^2 + f_n^2.$

   (c) $f_{2n} = f_{n+1}^2 - f_{n-1}^2.$

4. The Lucas sequence is given by

$$L_1 = 1, \quad L_2 = 3, \quad L_n = L_{n-1} + L_{n-2},\ n \geq 3.$$

   (a) Find the first 6 values of the Lucas sequence.

   (b) What should L_0 be defined to be to not mess up the recursion?

   (c) Use induction to prove that

$$L_n = f_{n-1} + f_{n+1}, \qquad \text{for } n \geq 1,$$

   where f_n is the nth Fibonacci number.

   (d) Prove that

$$L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n.$$