

Math 2001: Homework P11

Due: November 20, 2013

1. From the book do problems:

- (a) 5.2: 6, 8
- (b) 5.4: 4, 5
- (c) 6.1: 5, 10, 12

2. Let A be a set with n elements.

- (a) How many reflexive relations are there on A ?
- (b) How many symmetric, reflexive relations are there on A ?
- (c) How many equivalence relations are there of A , if $n = 5$?

3. Let R_n be the set of ways to place n non-attacking rooks on an $n \times n$ chess-board.

(a) Let $f : R_n \rightarrow \mathbb{Z}$ be given by

$$f(r) = \text{number of rooks on the diagonal squares of } r, \quad \text{for } r \in R_n.$$

For example, if $n = 4$,

$$f \left(\begin{array}{|c|c|c|c|} \hline \mathbb{R} & & & \\ \hline & * & & \mathbb{H} \\ \hline & & \mathbb{H} & \\ \hline & \mathbb{H} & & * \\ \hline \end{array} \right) = 2, \quad \text{where} \quad \begin{array}{|c|c|c|c|} \hline \mathbb{H} & & & \\ \hline & & & \mathbb{H} \\ \hline & & \mathbb{H} & \\ \hline & \mathbb{H} & & \\ \hline \end{array} \in R_4,$$

and I've marked the diagonal squares with $*$.

- i. What is $f(R_n)$?
 - ii. Is f injective?
 - iii. Is f surjective?
 - iv. Find $|f^{-1}(k)|$ for all $k \in f(R_4)$.
- (b) Find an injective function $g : R_n \rightarrow \mathbb{Z}$ (without changing the sets R_n and \mathbb{Z}).