Math 2001: Homework P3

Due: September 16, 2009

1. Identify whether each of the following statements is true or false. If it is true, prove it. If it is false, then find a counterexample.

   (a) Let A, B, C be sets. Then
   $$(A \cap B) \cup C = A \cap (B \cup C).$$

   (b) If $a, b \in \mathbb{Z}_{\geq 1}$ and both $\sqrt{a}$ and $\sqrt{b}$ are irrational, then $\sqrt{ab}$ is irrational.

2. From the book do problems:

   (a) 2.1.6, 2.1.11, 2.1.28

3. A point (m, n) in $\mathbb{R}^2$ is a *lattice point* if both $m, n \in \mathbb{Z}$.

   (a) Prove that the number of lattice points inside any circle centered at the origin is a number of the form $4k + 1$ for some integer k (This is 2.1.2 in the text).

   (b) What kind of proof did you use (indirect/direct, constructive/nonconstructive, etc.)?

   Hint: For (a), split the set of lattice points into subsets, depending on the quadrants.

4. Prove that $\sqrt{6}$ is irrational.