Math 2001: Homework P12

Due: December 9, 2009

1. From the book, do problems:
   (a) 2.2: 12 (be sure to use induction on (c)), 13

2. For each of the following sequences,
   • Give a formula for the $n$th term in the sequence,
   • Give a recursive definition for the sequence (ie. initial values and a recursive equation).
   (a) \{1, 2, 3, 4, 5, \ldots\}
   (b) \{1, 2, 4, 8, 16, 25, \ldots\}
   (c) \{1, 2, 6, 24, 120, \ldots\}

3. Let $f_0, f_1, \ldots$ be the Fibonacci sequence. For each of the following
   • Decide whether the identity is easier to prove by induction or directly using Binet’s formula (and some algebra). Explain.
   • Prove the identity using your preferred method.
   (a) $\sum_{k=0}^{n} f_k = f_{n+2} - 1$.
   (b) $f_{2n+1} = f_{n+1}^2 + f_n^2$.
   (c) $f_{2n} = f_{n+1}^2 - f_{n-1}^2$.

4. The Lucas sequence is given by
   \[ L_1 = 1, \quad L_2 = 3, \quad L_n = L_{n-1} + L_{n-2}, \quad n \geq 3. \]
   (a) Find the first 6 values of the Lucas sequence.
   (b) What should $L_0$ be defined to be to not mess up the recursion?
   (c) Use induction to prove that
   \[ L_n = f_{n-1} + f_{n+1}, \quad \text{for } n \geq 1, \]
   where $f_n$ is the $n$th Fibonacci number.
   (d) Prove that
   \[ L_n = \left( \frac{1 + \sqrt{5}}{2} \right)^n + \left( \frac{1 - \sqrt{5}}{2} \right)^n. \]