Math 2001: Homework P11

Due: November 18, 2009

1. From the book do problems:

    (a) 5.4: 4, 5

    (b) 6.1: 5, 10, 12

    (c) 6.2: 1, 3, 6

2. Let R_n be the set of ways to place n non-attacking rooks on an $n \times n$ chess-board.

    (a) Let $f : R_n \rightarrow \mathbb{Z}$ be given by

    $$f(r) = \text{number of rooks on the diagonal squares of } r, \qquad \text{for } r \in R_n.$$

    For example, if $n = 4$,

    $$f\left(\begin{array}{|c|c|c|c|} \hline \rook & & & \\ \hline & * & & \rook \\ \hline & & \rook & \\ \hline & \rook & & * \\ \hline \end{array} \right) = 2, \qquad \text{where} \qquad \begin{array}{|c|c|c|c|} \hline \rook & & & \\ \hline & & & \rook \\ \hline & & \rook & \\ \hline & \rook & & \\ \hline \end{array} \in R_4,$$

    and I've marked the diagonal squares with $*$.

    i. What is $f(R_n)$?

    ii. Is f injective?

    iii. Is f surjective?

    iv. Find $|f^{-1}(k)|$ for all $k \in f(R_4)$.

    (b) Find an injective function $g : R_n \rightarrow \mathbb{Z}$ (without changing the sets R_n and $\mathbb{Z}$).