Math 2001: Homework P3

Due: September 17, 2008

Give complete justifications for all your answers.

Problem 1

Prove the following (from the book)

1. \[ \frac{n}{n+1} = \sum_{k=1}^{n} \frac{1}{k(k+1)} . \]

2. \[ 2^n > n \text{ for all } n \in \mathbb{Z}_{\geq 0}. \]

3. \[ n! > 2^n \text{ for all } n \geq 4. \]

4. \[ \left( \frac{n}{2} \right)^2 = \sum_{k=0}^{n-1} k^3. \]

Problem 2

It can be shown that

\[ (X + Y + Z)^n = \sum_{k=0}^{n} \sum_{j=0}^{n-k} \binom{n}{k,j,n-k-j} X^k Y^j Z^{n-k-j} \]

(for a real challenge try proving it yourself, but this is not required for this assignment).

1. What does this say when \( X = Y = Z = 1 \)?

2. What does it say when \( Z = 0 \)?

Problem 3

Consider the following

Claim. The number \( n(n+1) \) is an odd number for every \( n \).

Proof. Assume the statement is true for \( n \). We prove the statement for \( n+1 \) by induction. Note that

\[ (n+1)((n+1)+1) = n(n+1) + 2(n+1). \]

By induction \( n(n+1) \) is odd. Thus, \( (n+1)((n+1)+1) \) is the sum of an odd number \( n(n+1) \) and an even number \( 2(n+1) \). The sum of an odd number and an even number is odd. Thus, we have proved the claim by induction.

I checked the claim and it doesn’t seem to work for \( n = 15 \), since \( 15 \cdot 16 = 240 \), which is even. What is wrong with the proof?