Math 2001: Homework P3

Due: September 17, 2008

Give complete justifications for all your answers.

Problem 1

Prove the following (from the book)

1. $\dfrac{n}{n+1} = \displaystyle\sum_{k=1}^{n} \frac{1}{k(k+1)}$.
2. $2^n > n$ for all $n \in \mathbb{Z}_{\geq 0}$.
3. $n! > 2^n$ for all $n \geq 4$.
4. $\dbinom{n}{2}^2 = \displaystyle\sum_{k=0}^{n-1} k^3$.

Problem 2

It can be shown that

$$(X+Y+Z)^n = \sum_{k=0}^{n} \sum_{j=0}^{n-k} \binom{n}{k, j, n-k-j} X^k Y^j Z^{n-k-j}$$

(for a real challenge try proving it yourself, but this is not required for this assignment).

1. What does this say when $X = Y = Z = 1$?
2. What does it say when $Z = 0$?

Problem 3

Consider the following

Claim. *The number $n(n+1)$ is an odd number for every n.*

Proof. Assume the statement is true for n. We prove the statement for $n+1$ by induction. Note that

$$(n+1)((n+1)+1) = n(n+1) + 2(n+1).$$

By induction $n(n+1)$ is odd. Thus, $(n+1)((n+1)+1)$ is the sum of an odd number $n(n+1)$ and an even number $2(n+1)$. The sum of an odd number and an even number is odd. Thus, we have proved the claim by induction. $\square$

I checked the claim and it doesn't seem to work for $n = 15$, since $15 \cdot 16 = 240$, which is even. What is wrong with the proof?