Math 2001: Homework 12

Due: December 3, 2008

Give complete justifications for all your answers.

Problem 1

1. Consider the sequence given by
   \[ a_n = 3a_{n-1} - 2a_{n-2}, \]  
   with \( a_1 = 3, a_2 = 7. \)
   Prove that \( a_n = 2^{n+1} - 1. \) Hint: Use induction.

2. Let \( A_1, A_2, \ldots, A_n \subseteq X \) be subsets of a big set \( X. \) Prove that
   \[ (A_1 \cap A_2 \cap \cdots \cap A_n)^c = A_1^c \cup A_2^c \cup \cdots \cup A_n^c \]
   in two ways: use both a proof by induction and a direct proof.
   Hint: For the direct proof, show that \( LHS \subseteq RHS \) and \( LHS \supseteq RHS. \)