Math 3130, Exam No. 1, Fall 2008

Friday, September 19, 2008 Instructor: S. Wu

I. (3 points) In each of the questions below, only one answer is correct. Circle the correct answer. You do not need to show the intermediate steps.

1. Which of the following matrices is in reduced row echelon form?

   A. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$; B. $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$; C. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$; D. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$.

2. If A, B, C are $n \times n$ matrices and $\vec{x}$, $\vec{y}$ are n-component column vectors, which of the following equalities is **not** always true?
   A. $(2A - B)\vec{x} = 2A\vec{x} - B\vec{x}$;
   B. $A(2\vec{x} - \vec{y}) = 2A\vec{x} - A\vec{y}$;
   C. $(AB)^{-1} = A^{-1}B^{-1}$ if A, B are invertible;
   D. $(AB)C = A(BC)$.

3. If A is an $n \times n$ matrix, which statement below is **not** equivalent to the other three?
   A. A is an invertible matrix;
   B. the equation $A\vec{x} = \vec{0}$ has a solution;
   C. the rank of A is n;
   D. the reduced row echelon for of A is the $n \times n$ identity matrix.

II. (4 points) Find the general solution of the system

$$\begin{aligned} x_1 - 2x_2 - x_3 + 3x_4 &= 0 \\ 3x_1 - 6x_2 - 6x_3 + 8x_4 &= 2. \\ -2x_1 + 4x_2 + 5x_3 - 5x_4 &= -1 \end{aligned}$$

III. (4 points) Let L be a line in $\mathbb{R}^3$ containing the vector $\vec{a} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$. Find the matrix of the linear transformation which is

1. the projection onto the line L;
2. the reflection about the line L.

IV. (4 points) For $A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 2 \\ -2 & 1 & -4 \end{bmatrix}$, use its reduced row echelon form to determine if A is invertible. If so, find A^{-1}.

Math 3130, Exam No. 2, Fall 2008

Friday, October 17, 2008 Instructor: S. Wu

I. (5 points) In each of the questions below, only one answer is correct. Circle the correct answer. You do not need to show the intermediate steps.

1. Suppose $A = [\vec{a}_1 \ \dots \ \vec{a}_n]$ is an $n \times n$ matrix. The statement that A is invertible is equivalent to each of the following **except**
   A. The system $A\vec{x} = \vec{0}$ is inconsistent;
   B. $\vec{a}_1, \dots, \vec{a}_n$ form a basis of $\mathbb{R}^n$;
   C. $\vec{a}_1, \dots, \vec{a}_n$ span $\mathbb{R}^n$;
   D. $\vec{a}_1, \dots, \vec{a}_n$ are linearly independent vectors.

2. If A is an $m \times n$ matrix, which of the following is **not** always true?
   A. $\operatorname{rank}(A) \le m$;
   B. $\operatorname{rank}(A) \le n$;
   C. $\dim \ker(A) \le m$;
   D. $\dim \ker(A) \le n$.

3. If two $n \times n$ matrices A and B are similar, then
   A. $AB = BA$;
   B. $\ker(A) = \ker(B)$;
   C. A and B have the same rank;
   D. There is only one invertible matrix S such that $S^{-1}AS = B$.

4. Which of the following sets is a linear space?
   A. The set of invertible 5×5 matrices;
   B. The set of non-invertible 5×5 matrices;
   C. The set of polynomials $p(x)$ of degree 5;
   D. The set of linear transformations from $\mathbb{R}^5$ to $\mathbb{R}^4$.

5. If T is a linear transformation from a linear space V to V, then
   A. the intersection of $\ker(T)$ and $\text{im}\,(T)$ must be $\{0\}$;
   B. the set of $f \in V$ such that $T(f) = f$ is a subspace of V;
   C. T is an isomorphism if $\ker(T) = \{0\}$;
   D. none of the above.

II. (5 points) Find a basis of the image of A and a basis of the kernel of A if

$$A = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 3 & 4 & -6 & 8 \end{bmatrix}.$$

III. (5 points) Let V be the linear subspace of $\mathbb{R}^{2\times 2}$ spanned by the matrices I_2 and $P = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$.

1. Is P^2 in V? (Justify your answer.) If so, find the coordinate vector $\left[P^2\right]_{\mathcal{B}}$, where $\mathcal{B} = (I_2, P)$;
2. Consider the linear transformation $T(M) = MP$ from V to V. Find the $\mathcal{B}$-matrix of T. Is T an isomorphism? (Justify your answer.)

Math 3130, Exam No. 3, Fall 2008

Wednesday, November 12, 2008 Instructor: S. Wu

I. (4 points) In each of the questions below, only one answer is correct. Circle the correct answer. You do not need to show the intermediate steps.

1. If A is an $n \times n$ matrix, which of the following is **incorrect**?
   A. $\det(A^T) = \det A$;
   B. $(A^T)^{-1} = (A^{-1})^T$;
   C. $(A^T)^2 = (A^2)^T$;
   D. $\text{rank}\,(A^T) + \text{rank}\,(A) = n$.
2. Let W be a subspace in $\mathbb{R}^n$ and let P be the matrix corresponding to the orthogonal projection onto W. Which of the following is correct?
   A. If $\vec{x}$ is in W, then $P\vec{x} = \vec{0}$;
   B. If $\vec{x}$ is in $W^\perp$, then $P\vec{x} = -\vec{x}$;
   C. The orthogonal projection onto $W^\perp$ is given by $I_n - P$;
   D. none of the above.
3. The determinant of the matrix $\begin{bmatrix} a & b & c \\ 1 & 1 & 7 \\ x & y & z \end{bmatrix}$ is equal to that of
   A. $\begin{bmatrix} c & a & b \\ 7 & 1 & 1 \\ z & x & y \end{bmatrix}$; B. $\begin{bmatrix} x & y & z \\ 1 & 1 & 7 \\ a & b & c \end{bmatrix}$; C. $\begin{bmatrix} a & b & c \\ 1 & 1 & 7 \\ 2x & 2y & 2z \end{bmatrix}$; D. $\begin{bmatrix} a-b & b-a & c \\ 0 & 0 & 7 \\ x-y & y-x & z \end{bmatrix}$.
4. Suppose $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ are in $\mathbb{R}^2$ and let $A = [\vec{v}_1\ \vec{v}_2\ \vec{v}_3]$. The area of the triangle whose vertices are $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ is
   A. $|\det A|$;
   B. $\sqrt{\det(A\,A^T)}$;
   C. $\sqrt{\det(A^T\,A)}$;
   D. none of the above.

II. (6 points) Let W be a subspace in $\mathbb{R}^4$ spanned by two vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$.

1. Find an orthonormal basis of W using the Gram-Schmidt process;
2. Find the QR decomposition of the matrix $A = [\vec{v}_1\ \vec{v}_2]$.

III. (5 points) Given three matrices $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, calculate

1. $\det\left((3A - 4C)^{-1}\,B^3\,(3A - 4C)^T\,B^{-1}\right)$;
2. $\det\begin{bmatrix} 2A & 0 \\ I_2 - B^T & C^{-1} \end{bmatrix}$;
3. $\det\begin{bmatrix} I_2 & A \\ B & C \end{bmatrix}$.