Solutions to Selected Exercises, Individual Homework #3

Assignment.
Section 1.5, Part 1: Linear functions and graphs (page 50): Exercises 4, 5(b).
Section 1.5, Part 2: Linear models (pages 51–52): Exercises 6, 7, 8.
Section 2.2 (pages 72–73): Exercises 1, 3, 4.

Section 1.5

4(b). \( y = -3x + 16 \).
5(b). \( x = -2. \ x = 2. \ x = -1/2. \ x = -487/2 \).
6. The formula is \( T = 0.05P \). The tax on a television set that costs $289.00 is \( T = 0.05(289.00) = 14.45 \). The tax on a toaster that costs $37.50 is \( T = 0.05(37.50) = 1.875 \) (or, rounded up to the nearest penny, $1.88).
7. Suppose \( W = 213 - 17Z \). If \( Z \) changes from 3 to 7, then \( W \) changes from 213–17(3) = 162 to 213 – 17(7) = 94. That is, \( W \) changes by –68 (or: \( W \) decreases by 68). If \( Z \) changes from 3 to 3.4, then \( W \) changes by –6.8. If \( Z \) changes from 3 to 3.02, then \( W \) changes by –0.34. Let \( \Delta Z \) denote a change in \( Z \), and \( \Delta W \) the change thereby produced in \( W \). Then \( \Delta W = -17\Delta Z \). That is, any change in \( Z \) produces –17 times as large a change in \( W \).
14(a). \( k = 1/2337 \) year\(^{-1} \) (or grams per year per gram). (b) We estimate that, after 40 years, about 0.0707755 grams remain.

Section 2.1

1(a). \( \Delta y/\Delta x = 8.2, 8.02, 8.002 \). (b). 8.
(c).
\[
\frac{\Delta y}{\Delta x} = \frac{f(2 + \Delta x) - f(2)}{\Delta x} = \frac{2(2 + \Delta x)^2 - 3 - (2 \times 2^2 - 3)}{\Delta x} = \frac{8 + 8\Delta x + 2(\Delta x)^2 - 3 - 5}{\Delta x} = \frac{8\Delta x + 2(\Delta x)^2}{\Delta x} = \frac{\Delta x(8 + 2\Delta x)}{\Delta x} = 8 + 2\Delta x.
\]
(d). \( f'(2) = \lim_{\Delta x \to 0} (8 + 2\Delta x) = 8 \). (d) \( y = f(2) + f'(2)(x - 2) = 5 + 8(x - 2) = 8x - 11 \).
7(a). \[
\frac{\Delta y}{\Delta x} = \frac{f(64 + \Delta x) - f(64)}{\Delta x} = \frac{\sqrt{64 + \Delta x} - \sqrt{64}}{\Delta x} = \frac{\sqrt{64 + \Delta x} - 8}{\Delta x}.
\]

(b).
\[
\frac{\Delta y}{\Delta x} = \frac{\sqrt{64 + \Delta x} - 8}{\Delta x} = \frac{\sqrt{64 + \Delta x} - 8}{\Delta x} \times \frac{\sqrt{64 + \Delta x} + 8}{\sqrt{64 + \Delta x} + 8} = \frac{(\sqrt{64 + \Delta x})^2 + 8\sqrt{64 + \Delta x} - 8\sqrt{64 + \Delta x} - 8^2}{\Delta x(\sqrt{64 + \Delta x} + 8)} = \frac{64 + \Delta x - 64}{\Delta x(\sqrt{64 + \Delta x} + 8)} = \frac{\Delta x}{\Delta x(\sqrt{64 + \Delta x} + 8)} = \frac{1}{\sqrt{64 + \Delta x} + 8}.
\]

(c).
\[
f'(64) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{1}{\sqrt{64 + \Delta x} + 8} = \frac{1}{\sqrt{64} + 8} = \frac{1}{8 + 8} = \frac{1}{16}.
\]

Section 2.2

1(a). \(f(x)\) looks more and more like a line. (b). Locally linear (or differentiable). (d). It looks roughly like the graph passes through the points (1.99, 54.2) (2.01, 57.8). This gives us a slope of \((57.8 - 54.2)/(2.01 - 1.99) = 180.0\). So we estimate that \(f'(2) \approx 180.0\).

4. The graph does not flatten out as we zoom in. This is because \(g(x)\) is not locally linear at \(x = 2\). It is locally linear at every other point, though. For example, if \(x > 2\), then the absolute values go away, so \(g(x) = x^2 - 10(x - 2) = x^2 - 10x + 20\), which is just a polynomial. If \(x < 2\), then \(|x - 2| = -(x - 2) = -x + 2\), so \(g(x) = x^2 - 10(-x + 2) = x^2 + 10x - 20\), which is another polynomial.

Section 2.3

6(a). \[
f'(x) = \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}
\]
\[
= \lim_{\Delta x \to 0} \frac{(\sqrt{x + \Delta x} - \sqrt{x})(\sqrt{x + \Delta x} + \sqrt{x})}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})}
\]
\[
= \lim_{\Delta x \to 0} \frac{(\sqrt{x + \Delta x})^2 - \sqrt{x}\sqrt{x + \Delta x} + \sqrt{x}\sqrt{x + \Delta x}}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})}
\]
\[
= \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})} = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})}
\]
\[
= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}.
\]
(b.) \( \sqrt{x} = x^{1/2} \) and \( 1/\sqrt{x} = x^{-1/2} \), so this exercise tells us that \( \frac{d}{dx}[x^{1/2}] = \frac{1}{2}x^{-1/2} \), which is the case \( p = 1/2 \) of the power formula.

12. (a) \( f'(x) = 21x^6 - 1.2x^3 + 3\pi x^2 \)

(b) \( \frac{d}{dx} \left[ \sqrt{3} \sqrt{x} + \frac{7}{x^5} \right] = \frac{\sqrt{3}}{2\sqrt{x}} - \frac{35}{x^6} \)

(c) \( h'(w) = \frac{2}{3}w^7 - \cos(w) - \frac{2}{3w^3} \)

(d) \( \frac{d}{du} \left[ \frac{4\cos(u)}{5} - \frac{3\tan(u)}{8} + \sqrt{u} \right] = -\frac{4}{5} \sin(u) - \frac{3}{8} \sec^2(u) + \frac{1}{3u^{2/3}} \)

(e) \( V'(s) = -\frac{1}{4s^{3/4}} \)

(f) \( F'(z) = \sqrt{7}\ln(2)2^z + \ln(1/2)(1/2)^z \)

(g) \( P'(t) = -at + v_0 \)

14(a). \( f(x) = x^{12} \). (c) \( f(x) = \sin(x) - \cos(x) \).