- General Notes -

1. Please make sure that your work flow is clear and easy to follow. There were a couple of people that had really messy work that was a bit difficult to follow and grade.

- Problem 1.5.7 -

1. Most people did pretty well on this. There was, however, a little bit of confusion regarding the sign of $\Delta W$. Most people got that $W(3) = 162$ and $W(7) = 92$, which does give us that $W$ decreases by 68 units. Therefore, we should get $\Delta W = -68$.

2. Be careful with your algebra. There were a couple of small mistakes in subtraction that ended up costing people points.

- Problem 2.1.1 -

1. I think that there was a little bit of confusion on how, exactly, we wanted you all to answer parts (b) and (d) of this problem, so I have provided a fully-worked solution below. Please take a look at it, and let me know if you have further questions.

**Q:** Let $f(x) = 2x^2 - 3$.

(a) Find the average rate of change $\frac{\Delta y}{\Delta x}$ of $f(x)$ with respect to $x$, from $x = 2$ to $x = 2 + \Delta x$, for each of the following three values of $\Delta x$: $\Delta x = 0.1$, $\Delta x = 0.01$, $\Delta x = 0.001$.

(b) Based on part (a) above, what might you guess $f'(2)$ is equal to?

(c) Use algebra to show that the average rate of change of $f(x)$ with respect to $x$, from $x = 2$ to $x = 2 + \Delta x$, is $8 + 2\Delta x$.

(d) Find the instantaneous rate of change of $f(x)$ at $x = 2$.

(e) Find the equation of the line tangent to the graph of $f(x)$ at $x = 2$.

**Solution:**

(a) We know that the average rate of change $\frac{\Delta y}{\Delta x}$ from $x$ to $x + \Delta x$ is given by the equation

$$\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$
So for $\Delta x = 0.1$ we have that
\[
\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{f(2 + 0.1) - f(2)}{0.1} = \frac{f(2.1) - f(2)}{0.1} = \frac{[2(2.1)^2 - 3] - [2(2^2) - 3]}{0.1} = \frac{[8.82 - 3] - [8 - 3]}{0.1} = \frac{5.82 - 5}{0.1} = \frac{0.82}{0.1} = 8.2
\]

Similar calculations for $\Delta x = 0.01$ and $\Delta x = 0.001$ should give us estimates of 8.02 and 8.002 respectively.

(b) Looking at the results of part (a), we can see that as $\Delta x$ gets closer and closer to 0, the average rate of change is getting closer and closer to 8. Since the value of $f'(2)$ is the limit as $\Delta x$ goes to 0 of $\Delta y/\Delta x$, we can guess that as $\Delta x$ goes to 0, $\Delta y/\Delta x$ will go to 8, giving us that $f'(2) \approx 8$ (I use the squiggly line here to denote that this is a guess).

(c) We know that the average rate of change of $f(x)$ at $x = a$ is given by
\[
\frac{\Delta y}{\Delta x} = \frac{f(a + \Delta x) - f(a)}{\Delta x}.
\]
Evaluating this, we get that
\[
\frac{\Delta y}{\Delta x} = \frac{f(2 + \Delta x) - f(2)}{\Delta x} = \frac{[2(2 + \Delta x)^2 - 3] - [2(2^2) - 3]}{\Delta x} = \frac{[8 + 8\Delta x + 2(\Delta x)^2 - 3] - 5}{\Delta x} = \frac{8\Delta x + 2(\Delta x)^2}{\Delta x} = \frac{\Delta x \cdot (8 + 2\Delta x)}{\Delta x} = 8 + 2\Delta x
\]

(d) We know that the instantaneous rate of change of $f(x)$ at $x = 2$ is the limit as $\Delta x$ goes to 0 of the average rate of change $\Delta y/\Delta x$ of $f(x)$ at $x = 2$. Well, in part (c) we computed that the average
rate of change of $f(x)$ at $x = 2$ was $8 + 2\Delta x$, so we have that

$$f'(2) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(2 + \Delta x) - f(2)}{\Delta x} = \lim_{\Delta x \to 0} 8 + 2\Delta x = 8$$

(e) To find the equation of the line tangent to the graph of $f(x)$ at $x = 2$ we only need to evaluate $f(2)$ and $f'(2)$. Well $f(2) = 5$, which means that the tangent line passes through the point $(2, 5)$ and $f'(2) = 8$ which means that the slope of the tangent line is $m = 8$. Using the point-slope form of a line, we get that the equation of the tangent line is

$$y = 8(x - 2) + 5$$

or equivalently

$$y = 8x - 11$$

- Problem 2.2.1 -

1. The graph was locally linear or differentiable. Just linear implies that the graph resembles a straight line no matter how far you zoom out.

2. The slope of the graph should have been 180, but I accepted anything reasonably close to that because eyeballing isn’t an exact science. This should have also been your estimate for $f'(2)$. 