CLS Solutions to Selected Exercises: Individual Homework Assignment #1


Exercise 14.

One stays infected for 14 days. This is because, as discussed in Section 1.2, the recovery coefficient $b$ equals the reciprocal of the number of days $k$ that one stays infected. Here we have $b = 1/14 \text{ day}^{-1}$, so $k = 14$ days.

Exercise 15.

$S_T = b/a = (1/14)/0.00001 = 7,142.9$ persons.

Exercise 16.

$S(1) \approx 4,444.6, I(1) \approx 2,903.4, R(1) \approx 2,650.0$; all units are persons.

Exercise 17.

$S(2) \approx 43,156.1, I(2) \approx 3,986.5, R(2) \approx 2,857.4$; all units are persons.

Exercise 18.

$S(3) \approx 41,435.7, I(3) \approx 5,422.1, R(3) \approx 3,142.1$; all units are persons.

Exercise 19.

$S(2) \approx 43,494.2, I(2) \approx 3,706.8, R(2) \approx 2800.0$; all units are persons.

Exercise 20.

(a) The new transmission coefficient is half the old one. So the new coefficient is $a = 0.5 \times 0.00001 = 0.000005$.

(b) $S_T = b/a = (1/14)/(0.000005) \approx 14,286$.

Another way to see this is: by cutting the transmission coefficient in half, we double the threshold value $S_T$ of $S$. (Think about why this makes intuitive sense.)
(c) At the outset of the disease, we have

\[
I'(0) = aS(0)I(0) - bI(0) = I(0)(aS(0) - b)
\]

\[
= (2100)(0.000005 \times 45400 - 1/14) = 326.7.
\]

That is, \(I'\) is initially positive, so \(I\) is initially increasing, so the quarantine does not eliminate the epidemic.

**Exercise 21.**

(a) Since \(b = 0.08 \text{ day}^{-1}\), we have that the infection lasts for \(k = 1/0.08 = 12.5\) days.

(b) We need \(I'\) to be positive; that is, \(aSI - bI > 0\). Factoring gives \(I(aS - b) > 0\). Since \(I > 0\), this means we need \(aS - b > 0\), or \(aS > b\), or \(S > b/a = 0.08/0.00002 = 4,000.0\). So there must be more than 4,000 susceptible individuals for the illness to take hold.

**Exercise 22.**

(a) Since the illness lasts for 4 days, we know that our recovery coefficient, \(b\), should be \(b = \frac{1}{4 \text{ days}} = 0.25 \frac{1}{\text{days}}\). Since a typical susceptible person meets only about 0.3% of infection population each day, we know that \(p = 0.3\% = .003\). Finally, since the infection is transmitted in only one contact out of 6, we know that \(q = \frac{1}{6} = 0.167\). Hence our SIR model for this measles-like disease looks like:

\[
S' = -aSI = -qpSI = -(0.167)(.003)SI = -0.0005SI
\]

\[
I' = aSI - bI = qpSI - bI = (0.167)(.003) - 0.25I = 0.0005SI - 0.25I
\]

\[
R' = bI = 0.25I.
\]

(b) We want to know: at what point does \(I\) start decreasing – that is, at what point does \(I'\) become negative? But

\[
I' = 0.0005SI - 0.25I
\]

\[
= I(0.0005S - 0.25).
\]

Since \(I \geq 0\) always, \(I'\) can only be negative if \(0.0005S - 0.25 < 0\), or \(0.0005S < 0.25\), or \(S < 0.25/0.0005 = 500\). In sum: if \(S < 500\), then \(I' < 0\), and hence the illness will fade away without becoming an epidemic.